A Property of Contradictions

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Content

Introduction

In classical propositional logic, a contradiction is an expression that is always false, regardless of the truth values of the propositions it contains. Example: "p and not p" is always false, regardless of whether the proposition p is true or false.

When generating contradictions algorithmically, it became apparent that contradictions always contain the operator "not".

That this is a generally valid theorem will be shown in the following chapter. My co-author Felix Hagen Z. Malsch found immediately a simple sketch of the proof.

We use the five classic operators according to the following table:

variable	variable	term	term	term	term	term
p	**q**	**p and q**	**p or q**	**p → q**	**p = q**	**not p**
Proposition p	Proposition q	p and q	p or q	p implies q	p equivalent q	not p
w	w	w	w	w	w	f
w	f	f	w	f	f	f
f	w	f	w	w	f	w
f	f	f	f	w	w	w

An example of a contradiction constructed with some of these operators is

$$(\text{not } (((p \rightarrow q) = p) \rightarrow q))$$

In an iterative process, more and more complex terms can be formed from elementary propositions.

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Theorem: Every contradiction contains at least one operator "not".

Proof: It has to be shown that a logical term which is constructed from the other four operators and any number of propositions without the operator "not" can never be a contradiction.

Logical terms are formed from operators and elementary propositions. More complex terms are formed by substituting proposition variables with terms. A contradiction is false for any combination of truth values of the propositions it contains. In particular, this also applies to the "worst case" that all propositions are true.

If we now consider the operators except "not", we see that in this case (first line of truth values in the table above) the terms are always true. Any combination and nesting does not change anything. If all propositions are true, we will never be able to construct a false proposition without the operator "not".

Therefore a contradiction must always contain the operator "not" at least once. **q.e.d.**