Calculation of the Optimized Zero-Moment Sc Airfoil Series

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1 Introduction

All airfoils can be described - according to Richard von Mises - by one formula.

"A much more general formula furnishing practically all kinds of profiles was proposed by R. v. Mises (1920). This formula is:" [1]

$$z_1 = z + \frac{a_1}{z} + \frac{a_2}{z^2} + ... + \frac{a_n}{z^n}$$

Hereby all variables designate complex numbers and z describes a circle centered on m_1 with radius 1 in the complex plane. This formula is a generalization of the Joukowski complex analytical function that describes the aerodynamic of an airfoil by the complex potential due to the conformal mapping of the circle in the complex plane onto the airfoil shape.

The determination of the airfoil for a certain combination of criteria like airfoil thickness, camber and moment coefficient is solved by the suitable choice of the complex constants a_i in the formula.

This document describes the calculation of the optimized airfoils of the Sc-series. This method was derived by a seven-fold iteration of the airfoil series. The iteration started with manually optimized airfoils of different camber and thickness. The iteration proceeded by regression of the complex parameters in relation to camber and thickness and the generation of a new series of airfoils that still needed manual enhancement. After seven iterations the calculation of the parameters of the series converged so that the airfoils show the desired properties.

Aerodynamic characteristics of airfoils of the optimized zero-moment Sc-series show:

- a high maximum lift coefficient, good for slow flying aircraft.
- a moment coefficient very close to zero, useful for flying wings.
- a good-natured behavior at higher angles of attack up to 20^o: The airfoil does not stall.
- a laminar flow up to angles of attack of about 10^o.
- a moment coefficient that stays close to zero and changes significantly only when the angle of attack exeeds 20^o.
- a sufficient lift and good performance even under icing conditions.
- a large airfoil area that permits the construction of big volume wings and facilitates internal reinforcement of the wing.

The airfoils of the Sc-series do not use the v.Mises-formula directly but are a kind of Theodorsen airfoil [2]. These are airfoils that provide a complex conformal mapping in two steps: The unit-circle is mapped onto an intermediate shape that is finally mapped onto the airfoil shape.

The Sc-series airfoils use a v.Mises airfoil as intermediate shape and map this onto the final Theodorsen airfoil which is here in fact also a v.Mises airfoil:

intermediate v.Mises Airfoil:

$$z_1 = z + \frac{b_1}{z} + \frac{b_2}{z^2} \tag{1}$$

Theodorsen Airfoil:

$$z_2 = z_1 + \frac{a_1}{z_1} + \frac{a_2}{z_1^2} + \frac{a_3}{z_1^3} \tag{2}$$

The airfoils of the optimized Sc-series are named according to the format **ScCDD**, where:

Sc designates the airfoil as a zero-moment airfoil from the Sc-series

C is the camber of the airfoil in percent

DD is the thickness of the airfoil in percent.

For example, Sc715 denotes the airfoil with 7% camber and 15% thickness.

2 Calculating the Parameters of the Complex Numbers

Airfoils of the Sc-series are functions of the desired camber c_5 and thickness d_5. c_5 and d_5 are as usual percentage numbers of the length of the airfoil.

After camber and thickness are defined an input correction has to be applied:

$$\Delta c = -604 \cdot \frac{c_5^2}{23233} + 349 \cdot \frac{c_5}{59317} - 1059 \cdot \frac{d_5}{42448} + \frac{44401}{45600}$$

$$\Delta d = 6085 \cdot \frac{c5}{28576} - 90 \cdot \frac{d_5^2}{16021} + 3313 \cdot \frac{d_5}{24395} - \frac{13751}{9200}$$

The corrected values c_4 for camber and d_4 for thickness are further used:

$$c_4 = c_5 + \Delta c$$

$$d_4 = d_5 + \Delta d$$

After the desired values are determined the airfoil is calculated according to the complex analytical functions (1) and (2) with the following parameters:

2.1 Unit-Circle

The unit-circle z with radius 1 is centered on the complex number m_1 as center:

$m_1 = re_{m_1} + i \cdot im_{m_1}$

where

$$re_{m_1} = -2 \cdot \frac{c_4^3}{27525} - 56 \cdot \frac{d_4}{7657}$$
$$im_{m_1} = c_4 \cdot (61 \cdot \frac{d_4}{61791} + \frac{1491}{61351}) + 110 \cdot \frac{d_4}{50169}$$

2.2 Intermediate v.Mises Airfoil

b_1 and b_2 are the parameters of the intermediate v.Mises airfoil z_1:

$b_1 = re_{b_1} + i \cdot im_{b_1}$

where

$$re_{b_1} = c_4 \cdot (10 \cdot \frac{d_4}{65149} - \frac{134}{48759}) - \frac{d4}{7805}$$
$$im_{b_1} = c_4 \cdot (17 \cdot \frac{d_4}{58583} - \frac{488}{52933}) - 23 \cdot \frac{d_4}{60742} + c_4 \cdot \frac{3134611 - 460020 \cdot d_4}{1629360172} + 6 \cdot \frac{d_4}{12545} - 2 \cdot \frac{c_5}{7381} - \frac{d_5}{47050}$$

$b_2 = re_{b_2} + i \cdot im_{b_2}$

where

$$re_{b_2} = c_4 \cdot (\frac{d_4^2}{249777} - \frac{d_4}{5947} + \frac{24}{13001}) - \frac{9}{15062}$$
$$im_{b_2} = 0$$

2.3 Theodorsen Airfoil

a_1 to a_3 are the parameters of the Theodorsen airfoil z_2:

$a_1 = re_{a_1} + i \cdot im_{a_1}$

where

$$re_{a_1} = -59 \cdot \frac{c_4^2}{33978} - 150 \cdot \frac{d_4}{9253} + 1$$
$$im_{a_1} = c_4 \cdot (52 \cdot \frac{d_4}{60713} - \frac{2601}{55096}) - 208 \cdot \frac{d_4}{60713}$$

$a_2 = re_{a_2} + i \cdot im_{a_2}$

where

$$re_{a_2} = c_4 \cdot (9 \cdot \frac{d_4}{63212} - \frac{49}{26015}) + 2 \cdot \frac{d_4}{36329}$$
$$im_{a_2} = 329 \cdot \frac{c_4}{24965} + 19 \cdot \frac{d_4}{7626}$$

$a_3 = re_{a_3} + i \cdot im_{a_3}$

where

$$re_{a_3} = -c_4^2 \cdot$$
$$\cdot \frac{636822100448135 \cdot d_4^3 - 22925618930680410 \cdot d_4^2 + 275111120245828206 \cdot d_4 - 1100428365754993940}{18785962845986379046710} +$$
$$-32 \cdot \frac{c_4}{53583} + \frac{d_4^2}{385903} - \frac{14}{41647}$$

3 Effect of the Parameters and Fine Tuning

The described complex conformal functions project the airfoil into the complex plane with the trailing edge pointing to the left (the negative real direction) and the trailing edge pointing to the right (the positive real direction).

Effect of the Parameters:

re_{m_1}: controls thickness distribution over the airfoil length (- sharpens trailing edge)

im_{m_1}: changes camber (+ positive camber)

re_{a_1}: adjusts thickness (Ellipse) (+ reduces thickness)

im_{a_1}: changes thickness distribution over the airfoil length and a little camber

re_{a_2}: bluntness of the leading edge (nose) (+ makes nose blunt, - sharpens trailing edge)

im_{a_2}: changes camber (+ negative camber)

re_{a_3}: changes thickness symmetrically (+ inflates center, sharpens edges; - deflates center, rounds edges)

im_{a_3}: controls S-curve of the centerline (+ reduces CM (lifts curve), - increases CM)

re_{b_1}: controls thickness distribution, parallelising of the upper- and lower side in the center (+ sharpens trailing edge, - inflates center)

im_{b_1}: adjusts S-curve of the centerline (- reduces CM, + increases CM (lowers curve))

re_{b_2}: changes thickness distribution; difficult: rounds leading edge and sharpens trailing edge (+ sharpens trailing edge, - distributes thickness over the airfoil length)

im_{b_2}: double S-curve (+ hump, - belly)

Symmetrical airfoils are achieved if the imaginary parts of all complex parameters are set to zero.

4 Example: Airfoil Sc715 Parameters

The following are the automatically generated parameters of the Sc715 airfoil with 7% camber and 15% thickness and the manually fine tuned parameters:

4.1 Automatically Generated Parameters

```
 Sc715 Airfoil
11-03-2013 (c) Forschungskontor, Dipl.-Ing.(FH) Kapt.(AG) Wolf Scheuermann
Aerodynamic - Conformal Mapping - Complex Potential
```

z : circle centered on m1 with radius 1

intermediate v.Mises Airfoil: $z_1 = z + \frac{b_1}{z} + \frac{b_2}{z^2}$

Theodorsen Airfoil: $z_2 = z_1 + \frac{a_1}{z_1} + \frac{a_2}{z_1^2} + \frac{a_3}{z_1^3}$

```
-0.13410 = Re(m1)
0.28830 = Im(m1)

-0.00410 = Re(b1)
-0.04640 = Im(b1)
0.00060 = Re(b2)
0.00000 = Im(b2)

0.67400 = Re(a1)
-0.26870 = Im(a1)
0.00310 = Re(a2)
0.12320 = Im(a2)
-0.00370 = Re(a3)
0.00000 = Im(a3)
```

4.2 Manually Fine Tuned Parameters

The parameters of the manually tuned and refined airfoil are:

```
Sc715 Airfoil
01-21-2009 (c) Forschungskontor, Dipl.-Ing.(FH) Kapt.(AG) Wolf Scheuermann
Aerodynamic - Conformal Mapping - Complex Potential
```

z : circle centered on m1 with radius 1

intermediate v.Mises Airfoil: $z_1 = z + \frac{b_1}{z} + \frac{b_2}{z^2}$

Theodorsen Airfoil: $z_2 = z_1 + \frac{a_1}{z_1} + \frac{a_2}{z_1^2} + \frac{a_3}{z_1^3}$

```
-0.13450 = Re(m1)
0.28880 = Im(m1)
```

```
-0.00410 = Re(b1)
-0.04650 = Im(b1)
0.00060 = Re(b2)
0.00000 = Im(b2)

0.67320 = Re(a1)
-0.26860 = Im(a1)
0.00320 = Re(a2)
0.12320 = Im(a2)
-0.00360 = Re(a3)
0.00000 = Im(a3)
```

4.3 Aerodynamic Properties

The aerodynamical check of the tuned airfoil according to the methods of Richard Eppler [3] reveals the following properties:

$$CA_{max} = 2.05217 \quad at \quad \alpha_{max} = 24.966^o$$
$$CD_{min} = 0.00755 \quad at \quad CA = 0.89012$$

$$thickness \quad d = 15.05\%$$
$$x_d = 23.8\%$$
$$camber \quad c = 7.036\%$$
$$x_c = 34.87\%$$
$$\alpha_0 = -2.013^o$$
$$CM_{0.25} = -5.399 \cdot 10^{-5}$$
$$d_{CA} = 7.163$$
$$x_{np} = 25.8\%$$
$$\alpha_e = 4.579^o$$
$$CA_e = 0.7233$$

4.4 Airfoil Sc715

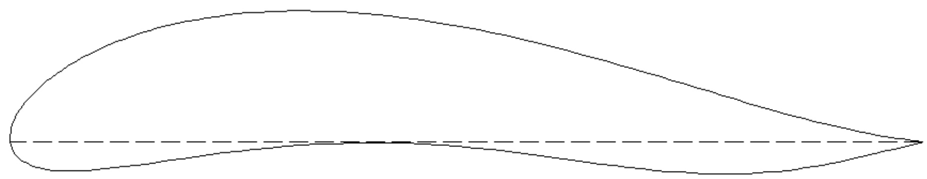

Figure 1: Airfoil Sc715

5 References

References

[1] Richard von Mises: Theory of Flight, p.128
Dover, New York 1959

[2] Holt Ashley, Martin Landahl: Aerodynamics of Wings and Bodies, p.54
Dover, New York 1985

[3] Frank Ranis: NURFLUEGEL.EXE
according to Michael Möller's Nur.exe and Wineppler.exe, based on the work of Dipl.-Ing. Jochen Schneider, Dipl.-Ing. Erich Truckenbrodt, Prof. Dr. R. Eppler, John G. Roncz's Eppler.exe, n.n. 2002