The Circling Paradoxon

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1 Introduction

This document describes a paradoxon in instrument flight procedures. Usually, timing of flight procedures depends on speed and altitude of the aircraft. Under certain conditions, independently of flight conditions a fixed timing can be used on downwind to commence the descend towards the runway.

2 The Circling Paradoxon

Circling Approach Timing

When on downwind of a Circling Approach the given rule for timing outbound from abeam threshold (THR) of the runway (RWY) is to fly outbound three seconds for every 100 ft height to loose (ΔH):

$$t_{out} = \Delta H_{[ft]} \cdot \frac{3\,sec}{100\,ft} = \Delta H_{[ft]} \cdot \frac{0.5\,min}{1000\,ft} = \frac{\Delta H_{[ft]}}{2000\,fpm}$$

After t_{out} the Top of Descend (TOD) is reached and the descend towards the RWY begins with a 180^o Standard Rate Turn (SRT):

$$t_{TOD} = t_{out}$$

To calculate the Rate of Descend (RoD) for turning base and to final we have $t_{in} = t_{out}$ and $t_{\frac{SRT}{2}} = 1\,min$. The Total Time t_{TTL} is

$$t_{TTL} = t_{out} + t_{Desc} = t_{out} + t_{\frac{SRT}{2}} + t_{in} = 2 \cdot t_{out} + 1\,min$$

with the Descend Time t_{Desc}

$$t_{Desc} = t_{\frac{SRT}{2}} + t_{in} = \frac{\Delta H_{[ft]}}{RoD_{[fpm]}}$$

Therefore

$$t_{TOD} = t_{out} = t_{TTL} - t_{Desc}$$

$$\Rightarrow \quad \frac{\Delta H_{[ft]}}{2000\,fpm} = 2 \cdot \frac{\Delta H_{[ft]}}{2000\,fpm} + 1\,min \cdot \frac{\Delta H_{[ft]}}{\Delta H_{[ft]}} - \frac{\Delta H_{[ft]}}{RoD_{[fpm]}}$$

Divided by ΔH:

$$\frac{1}{2000\,fpm} = \frac{1}{1000\,fpm} + \frac{1\,min}{\Delta H_{[ft]}} - \frac{1}{RoD_{[fpm]}}$$

$$\Rightarrow \quad \frac{1}{2000\,fpm} - \frac{1}{1000\,fpm} - \frac{1\,min}{\Delta H_{[ft]}} = -\frac{1}{RoD_{[fpm]}}$$
$$\Rightarrow \quad \frac{1}{1000\,fpm} - \frac{1}{2000\,fpm} + \frac{1\,min}{\Delta H_{[ft]}} = \frac{1}{RoD_{[fpm]}}$$
$$\Rightarrow \quad \frac{2-1}{2000\,fpm} + \frac{1}{\Delta H_{[fpm]}} = \frac{1}{RoD_{[fpm]}}$$
$$\Rightarrow \quad \frac{\Delta H_{[fpm]} + 2000\,fpm}{\Delta H_{[fpm]} \cdot 2000\,fpm} = \frac{1}{RoD_{[fpm]}}$$

and so we get finally:

$$RoD_{[fpm]} = \frac{2000 \cdot \Delta H_{<ft>}}{2000 + \Delta H_{<ft>}} [fpm]$$

where the brackets <> mean to enter just the quantity, without the unit. With this formula the RoD can be calculated according to the downwind height.

If, on the other hand, a fixed $RoD = 1000\,fpm$ is used the formula for t_{out} provides us with

$$\Delta H_{[ft]} = 2000\,fpm \cdot t_{out_{[min]}}$$

Inserted into the formula for RoD results in

$$1000\,fpm = \frac{2000\,fpm \cdot 2000\,fpm \cdot t_{out_{<min>}}}{2000\,fpm + 2000\,fpm \cdot t_{out_{<min>}}}$$
$$\Rightarrow \quad 1000\,fpm = 2000\,fpm \cdot \frac{t_{out_{<min>}}}{1 + t_{out_{<min>}}}$$
$$\Rightarrow \quad \frac{1}{2} \cdot (1 + t_{out_{<min>}}) = t_{out_{<min>}}$$
$$\Rightarrow \quad \frac{1}{2} + \frac{1}{2} \cdot t_{out_{<min>}} = t_{out_{<min>}}$$
$$\Rightarrow \quad \frac{1}{2} = t_{out_{<min>}} - \frac{1}{2} \cdot t_{out_{<min>}} = \frac{1}{2} \cdot t_{out_{<min>}}$$
$$\Rightarrow \quad \frac{1}{2} \cdot t_{out_{[min]} = \frac{1}{2}\,min}$$
$$\Rightarrow \quad t_{out} = 1\,min$$

This describes the following procedure ("**Circling Paradoxon**"):

1. Fly the circling approach downwind from abeam THR for one minute straight and level, then
2. start the descend with Standard Rate Turn (SRT) and a constant $RoD = 1000\, fpm$.
3. Rolling out on final, this RoD leads directly to the THR of the RWY.

Note: *This procedure is independent of speed and height of the A/C.*

Note: *The downwind distance abeam to the RWY has to be the actual diameter of the SRT.*

Therefore, approaching the RWY on the opposite course half a 80^o-Procedure Turn (P/T) should be flown until the downwind course is intercepted.

60 seconds after abeam passing the threshold the descend can be started with $RoD = 1000\, fpm$, independent from circling ALT. The TOD on downwind is always fix at 60 seconds after the abeam threshold position.

Note: *The formulas do not take the wind into account and are therefore rules of thumb.*

3 Literature

References

[1] Wolf Scheuermann: Instrument Flight Procedures, v1.3.
Lufthansa Flight Training (LFT) GmbH,
Bremen 2015