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ABSTRACT

Pulse shapes of currents in proportional counters are calculated for cases where the incident particle might pass at different distances from the wire. The pulse propagation along a resistive wire is treated with the help of Laplace transforms. The current as well as the collected charge on the ends of the counter are given. This work differs from the excellent treatment of the problem by Radeka by deriving compact analytic expressions for current rise and charge division of the appropriate pulse shapes, which can be handled even on a pocket calculator. Finally, the dynamic input resistances of amplifiers, which influence the precision of charge measurement, are evaluated.

DOC/kw – rm – db – msv – mg

CONTENTS

1. CURRENT DYNAMICS OF PROPORTIONAL COUNTERS

The counter should be of square cross-section and the electron drift velocity v_e constant. A particle should traverse the counter at $t = 0$, as indicated in Fig. 1, and produce a uniform ionization along its trajectory. If the total

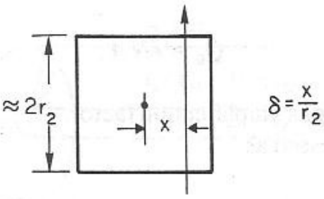


Fig. 1

number of ion pairs is N, we obtain for the differential number ΔN of electrons arriving on the wire within the time interval $\Delta\tau$ at time τ:

$$\Delta N = 0 \qquad \text{for} \quad \tau \leq \frac{x}{v_e} = \tau_1 \ ,$$

$$= \frac{N}{r_2} v_e^2 \frac{\tau\Delta\tau}{\sqrt{v_e^2\tau^2 - x^2}} \qquad \text{for} \quad \tau_1 \leq \tau \leq \frac{\sqrt{x^2 + r_2^2}}{v_e} = \tau_2 \ ,$$

$$= 0 \qquad \text{for} \quad \tau \geq \tau_2 \ .$$

It is assumed that the multiplication should take place on the wire surface. The electrons are instantaneously bound, whereas the positive ions will move towards the cathode.

A charge ΔQ, moving in an electric field E with velocity v, induces at the electrodes a current

$$\Delta I = \frac{\Delta Q}{U} E\mathrm{v} \ ,$$

where U is the voltage between the electrodes.

Apart from the corners of the tube, the electric field is sufficiently well approximated by

$$E = \frac{U}{\ln(r_2/r_1)} \frac{1}{r} \ ,$$

where r_1 is the wire radius and r_2 the "radius" of the tube.

The ion velocity should be given by

$$\mathrm{v} = b \cdot E$$

(b is the ion mobility).

Using the last three expressions, we obtain:

$$\Delta I(t, \tau) = 0 \qquad \text{for } t < \tau \ ,$$

$$= \frac{\Delta Q}{2\ln(r_2/r_1)} \frac{1}{t + t_1 - \tau} \qquad \text{for } t \geq \tau \ ,$$

where

$$t_1 = \frac{r_1^2 \ln(r_2/r_1)}{2bU} \ .$$

Typical values are

$$r_2/r_1 = 200; \quad U = 1.5\,\text{kV}; \quad b = 1\,\text{cm}^2/\text{V}\cdot\text{s};$$

$$t_1 \approx 10^{-8}\,\text{s}.$$

The ion charge ΔQ is related to its producing electron number ΔN by

$$\Delta Q = e \Delta N A$$

and the total charge by

$$Q_0 = eNA ,$$

where e is the elementary charge and A is the gas amplification factor.

The induced current can finally be expressed as

$$
\begin{aligned}
I &= 0 && \text{for} \quad t \leq \tau_1 \;, \\
&= \frac{Q_0 v_e^2}{2r_2 \ln(r_2/r_1)} \int_{\tau_1}^{t} \frac{\tau d\tau}{(t + t_1 - \tau)\sqrt{v_e^2\tau^2 - x^2}} && \text{for} \quad \tau \leq t \;, \\
&= \frac{Q_0 v_e^2}{2r_2 \ln(r_2/r_1)} \int_{\tau_1}^{\tau_2} \frac{\tau d\tau}{(t + t_1 - \tau)\sqrt{v_e^2\tau^2 - x^2}} && \text{for} \quad t \geq \tau_2 \;, \\
&= 0 && \text{for} \quad t \geq t_2 \;,
\end{aligned}
$$

where

$$t_2 \approx \frac{r_2^2 \ln(r_2/r_1)}{2bU}$$

(typical value: $t_2 \approx 1.7 \times 10^{-3}$ s, for $r_2 = 1$ cm).

Figure 2 shows current shapes, with δ (distance of passage) and t_1 (wire radius, ion mobility) as parameters, and the corresponding curves for the collected charges.

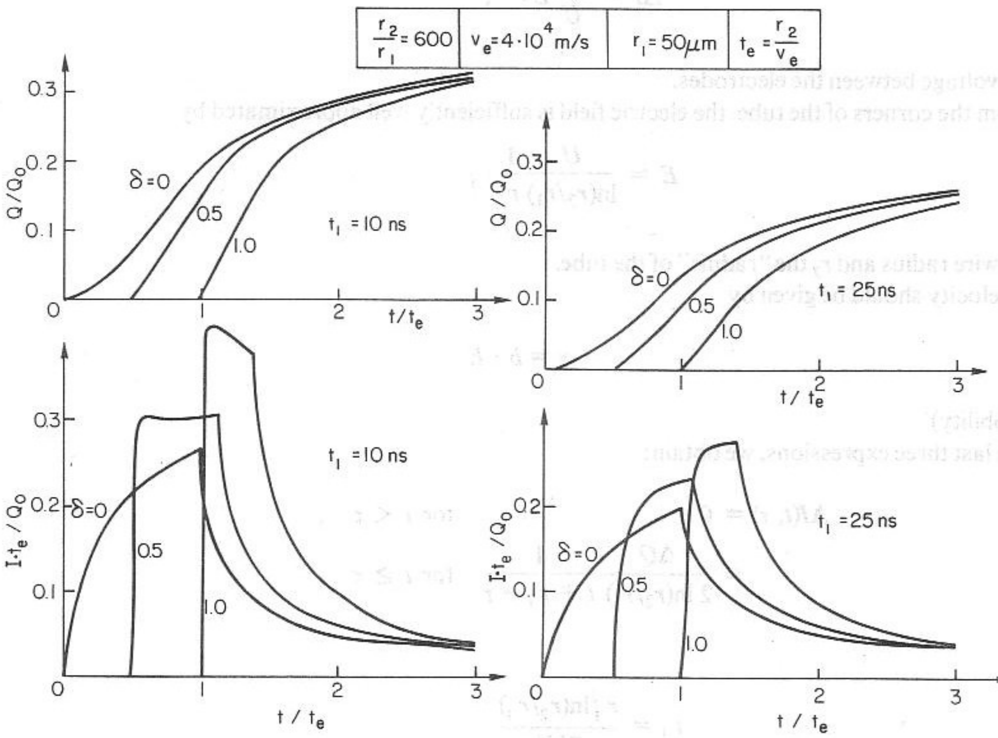


Fig. 2

A precise relative measurement of the charge can be made only after a time which is much larger than $3t_e \approx 1$ μs. This is due to the uncertainty of the time at which the particle has passed the counter and to the finite settling time of the amplifiers.

2. CHARGE DIVISION ON A PROPAGATION LINE WITH LOSSES

2.1 Dispersion relation and wave propagation

First of all we have to establish the dispersion relation for such a line, which will be calculated for a coaxial cable with a resistive inner conductor and a loss-free outer conductor. The necessary notation is given in Fig. 3.

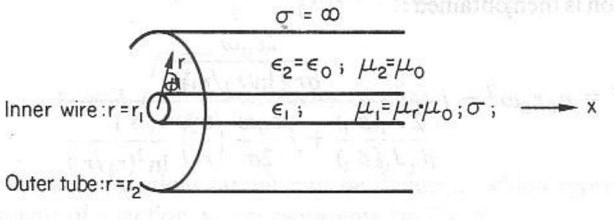


Fig. 3

A solution of Maxwell's equations, which is independent of ϕ and has a propagation term $p = \exp j \cdot (\omega t - hx)$, is envisaged. We have to deal with the three components

$$E_x(r) \cdot p \; ; \quad E_r(r) \cdot p \; ; \quad H_\phi(r) \cdot p \; .$$

Introducing the dimensionless variables

$$\bar{\rho} = \sqrt{k_1^2 - h^2}\, r \; ; \quad \rho = \sqrt{k_2^2 - h^2}\, r$$

with

$$k_1^2 = \varepsilon_0 \mu_1 \omega^2 - j\mu_1 \sigma\omega \quad \text{(inside the wire)}$$
$$\varepsilon_1 = \varepsilon_0 - j\sigma/\omega$$
$$k_2^2 = \varepsilon_0 \mu_0 \omega^2 \qquad \text{(interspace)}$$

we obtain from the continuity condition for E_x at $r = r_1$ and the boundary condition $E_x = 0$ on $r = r_2$:

$$E_{x_1} = A_1 J_0(\bar{\rho}_1) = E_{x_2} = A_2 \left[J_0(\rho_1) - \frac{J_0(\rho_2)}{Y_0(\rho_2)} Y_0(\rho_1) \right]$$

(J and Y are Bessel and Neumann functions).

The continuity condition

$$H_{\phi_1}(r_1) = H_{\phi_2}(r_1)$$

leads finally to the following condition:

$$\frac{\sqrt{\varepsilon_1/\mu_1}\, k_1}{\sqrt{k_1^2 - h^2}} \frac{J_1(\bar{\rho}_1)}{J_0(\bar{\rho}_1)} = \frac{\sqrt{\varepsilon_2/\mu_2}\, k_2}{\sqrt{k_2^2 - h^2}} \frac{Y_0(\rho_2) J_1(\rho_1) - J_0(\rho_2) Y_1(\rho_1)}{Y_0(\rho_2) J_0(\rho_1) - J_0(\rho_2) Y_0(\rho_1)}$$

from which the dispersion relation can be extracted.

Introducing the restriction conditions

$$\omega_1 \ll \sigma/\varepsilon_0 \quad [\omega \leq 10^{16}\ \text{Hz for } \sigma = 5 \times 10^6\ (\Omega \cdot \text{m})^{-1}]$$
$$|h| \ll \sigma/\varepsilon_0$$

we obtain the following simplifications:

$$\sqrt{k_1^2 - h^2} \approx \sqrt{-j\mu_1\sigma\omega} \quad , \qquad \bar{\rho}_1 \approx r_1 \sqrt{-j\mu_1\sigma\omega} \quad ,$$

and for

$$\omega_2 \leq \sqrt{\frac{2}{\mu_2 \varepsilon_2}} \frac{1}{r_2} \qquad [\omega \leq 8 \times 10^{10}\ \text{Hz for } r_2 = 5 \times 10^{-3}\ \text{m}]$$

the fraction containing the Bessel and Neumann functions can be approximated by

$$\frac{1}{\rho_1 \ln(r_2/r_1)}\left[1-\frac{\rho_2^2}{4\ln(r_2/r_1)}\right].$$

The following dispersion relation is then obtained:

$$h^2=\mu_0\varepsilon_0\omega^2-j\,\frac{\dfrac{2\varepsilon_0\omega}{\sigma r_1^2\ln(r_2/r_1)}}{\dfrac{2J_1(\bar\rho_1)}{\bar\rho_1 J_0(\bar\rho_1)}+j\dfrac{\varepsilon_0\omega}{2\sigma}\left(\dfrac{r_2}{r_1}\right)^2\dfrac{1}{\ln^2(r_2/r_1)}}$$

For

$$\omega \geq \frac{2}{\mu_1\sigma r_1^2} \qquad [\omega \geq 10^9\,\text{Hz for } r_1 = 20\,\mu\text{m}\,,\ \mu_1=\mu_0,\ \sigma = 5\times 10^6\,(\Omega\cdot\text{m})^{-1}]\quad ,$$

J_1/J_0 can be replaced by its asymptotic value, namely $-j$; thus the first term of the denominator is much larger than the second one (for $\omega \leq \omega_2$). The high-frequency approximation of the dispersion relation becomes

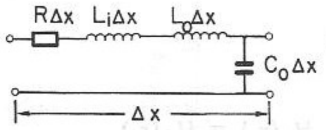
$$h_h^2=\mu_0\varepsilon_0\omega^2-\frac{\varepsilon_0}{r_1\ln(r_2/r_1)}\sqrt{-j\frac{\mu_1}{\sigma}\omega^3}$$

for

$$\frac{2}{\mu_1\sigma r_1^2}\leq\omega\leq\sqrt{\frac{2}{\mu_2\varepsilon}\frac{1}{r_2}}\ .$$

The second term, divided by the capacity per metre of the line, represents Rayleigh's impedance of a free wire in the high-frequency approximation;

From h_h an equivalent circuit can be deduced, where the lumped element of a section Δx is represented as shown in Fig. 4.



$$C_0=\frac{2\pi\varepsilon_0}{\ln(r_2/r_1)}$$: capacity per metre of cable interspace

$$L_0=\frac{\mu_0}{2\pi}\ln(r_2/r_1)$$: inductance per metre of cable interspace

$$L_i=\frac{1}{2\pi r_1}\sqrt{\frac{\mu_1}{2\sigma\omega}}$$: inner inductance per metre of wire

$$R=\frac{1}{2\pi r_1}\sqrt{\frac{\mu_1\omega}{2\sigma}}$$: resistance per metre of wire

Fig. 4

In order to obtain a handsome expression for the dispersion relation for "low" frequencies, the Bessel functions are approximated by a Taylor series, including terms of $\bar\rho_1^3$, i.e. the following restriction has to be fulfilled:

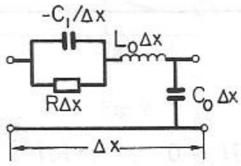
$$\omega\leq\frac{2}{\mu_1\sigma r_1^2}\qquad \left[\omega\leq 10^9\,\text{Hz for } r_1=20\,\mu\text{m}\,,\ \sigma=5\times10^6\,(\Omega\cdot\text{m})^{-1}\mu_1=\mu_0\right]\ .$$

The calculation yields

$$h_l^2 = \varepsilon_0\mu_0\omega^2 - j\frac{\left[2\varepsilon_0/\sigma r_1^2\ln(r_2/r_1)\right]\omega}{1 - j\frac{\mu_0\sigma r_1^2}{8}\left\{\mu_r - \left[\frac{2}{\mu_0\sigma c}\frac{r_2}{r_1^2\ln(r_2/r_1)}\right]^2\right\}\omega} \quad .$$

$$c = \sqrt{\frac{1}{\varepsilon_0\mu_0}} = \text{velocity of light}$$

From this dispersion relation an equivalent circuit can be deduced, which represents a refined version of the Thomson cable. The lumped elements of a section Δx are represented in Fig. 5.



$C_0 = \frac{2\pi\varepsilon_0}{\ln(r_2/r_1)}$: capacity per metre of cable interspace

$L_0 = \frac{\mu_0}{2\pi}\ln(r_2/r_1)$: inductance per metre of cable interspace

$R = \frac{1}{\pi\sigma r_1^2}$: d.c. resistance per metre of inner conductor

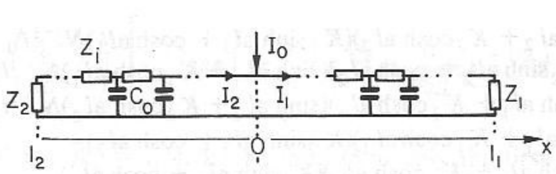
$C_l = \frac{\pi\mu_0\sigma^2 r_1^4}{8}\left\{\mu_r - \left[\frac{2}{\mu_0\sigma c}\frac{r_2}{r_1^2\ln(r_2/r_1)}\right]^2\right\}$: not interpretable

Fig. 5

Separating the real and the imaginary part of h_l and neglecting terms that are compatible with the outlined restrictions leads to

$$h_l^2 = \varepsilon_0\mu_0\left(1 + \frac{1}{4\ln(r_2/r_1)}\left\{\mu_r - \left[\frac{2}{\mu_0\sigma c}\frac{r_2}{r_1^2\ln(r_2/r_1)}\right]^2\right\}\right)\omega^2 - j\frac{2\varepsilon_0}{\sigma r_1^2\ln(r_2/r_1)}\omega \quad .$$

The equivalent circuit of a counter can now be established as shown in Fig. 6.



I_0: injected current, originating from the discharge
I_1; I_2: propagating currents in the line
Z_i: Z_h or Z_l, depending on the high- or low-frequency model
Z_1; Z_2: terminations of the line

Fig. 6

Next, the differential equations of such a line are worked out as well as their Laplace transforms (i.e. in the dispersion relations $j\omega$ has to be replaced by s).

Original function	Laplace transform
$F(x, t)$	$f(x,s)$
$\frac{\partial I}{\partial x} = C_0 \frac{\partial U}{\partial t}$	$i_x = sC_0 u$
$\frac{\partial U}{\partial x} = Z_i I$	$u_x = z_i(s) i$
$U(x, +0) = 0$	
$U_t(x, +0) = 0$	
$I(x, +0) = 0$	
$I_t(x, +0) = 0$	
$I(0, t) = I_0 = I_1 - I_2$	$i_0(s) = i_1(0, s) - i_2(0, s)$

There then follows the differential equation:

$$i_{xx} - a_i^2 i = 0 \quad *, \qquad i: h, l$$

$$i_\nu = A_\nu \cosh a_i x + B_\nu \sinh a_i x \qquad \nu: 1, 2$$

where

$$a_h^2 = \mu_0 \varepsilon_0 s^2 + \frac{\varepsilon_0}{r_1 \ln(r_2/r_1)} \sqrt{\frac{\mu_1}{\sigma}}\, s^{3/2} = \bar{\beta}^2 s^2 + \bar{\alpha} s^{3/2}$$

$$a_l^2 = \varepsilon_0 \mu_0 \left(1 + \frac{1}{4 \ln(r_2/r_1)} \left\{ \mu_r - \left[\frac{2}{\mu_0 \sigma c} \frac{r_2}{r_1^2 \ln(r_2/r_1)} \right]^2 \right\} \right) s^2 + \frac{2 \varepsilon_0}{\sigma r_1^2 \ln(r_2/r_1)} s = \beta^2 s^2 + \alpha s \quad .$$

From the initial conditions above and from the boundary conditions

$$z_1 = \frac{u_1(l_1, s)}{i_1(l_1, s)} \; ; \; z_2 = \frac{u_2(-l_2, s)}{i_2(-l_2, s)} \; ,$$

we obtain

$$B_1 = B_2 = B$$

and

$$\begin{aligned}
A_1 &= (\sinh a l_2 + K_2 \cosh a l_2)(K_1 \sinh a l_1 + \cosh a l_1) N^{-1} i_0 \\
A_2 &= -(K_2 \sinh a l_2 + \cosh a l_2)(\sinh a l_1 + K_1 \cosh a l_1) N^{-1} i_0 \\
B &= -(\sinh a l_1 + K_1 \cosh a l_1)(\sinh a l_2 + K_2 \cosh a l_2) N^{-1} i_0 \\
N &= (\sinh a l_1 + K_1 \cosh a l_1)(K_2 \sinh a l_2 + \cosh a l_2) \\
&\quad + (\sinh a l_2 + K_2 \cosh a l_2)(K_1 \sinh a l_1 + \cosh a l_1)
\end{aligned}$$

where

$$K_\nu = \frac{sC_0}{a} Z_\nu \; .$$

(The index i of a has been ignored)

The currents on the counter ends are

$$i_\nu = A_\nu \cosh a l_\nu + B \sinh a l_\nu = T r_\nu i_0 \quad .$$

If the ends are short circuited, i.e. $z_v = 0$, the following simplified formulae result:

$$i_1 = \frac{\sinh al_2}{\sinh al} i_0 \quad ; \quad i_2 = \frac{\sinh al_1}{\sinh al} i_0 \quad ; \quad \text{where} \quad l = l_1 + l_2 \quad .$$

We are interested in the instantaneous current rise and the collected charge after long periods on the extremities of the counter. It is sufficient to know the initial rise and the decay behaviour of the injected current. The following current shape (Fig. 7) (see Fig. 2) is assumed:

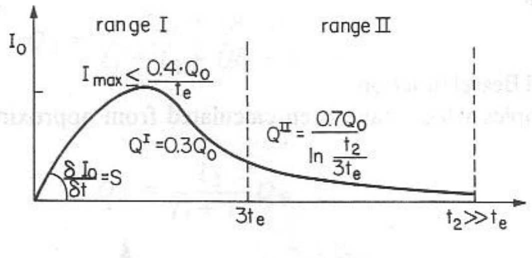


Fig. 7

range I:

$$I^{\mathrm{I}}(t) = S \cdot t + \text{(higher terms)} \qquad i_0^{\mathrm{I}}(s) = S \frac{1}{s^2} + \cdots$$

range II:

$$I^{\mathrm{II}} = \frac{0.7Q_0}{\ln(t_2/3t_e)} \frac{1}{t} \qquad i_0^{\mathrm{II}} = Q^{\mathrm{II}} \int_{3t_e}^{t_2} \frac{e^{-st}}{t} \, dt \, .$$

2.2 Current rise on the end of the counter

The case of the short-circuited line will be considered. Tr is developed into a series of exponential functions:

$$Tr = e^{-al_1} + e^{-a(l+2l_2)} + \cdots \quad .$$

Only the first term is essential, because the contributions of all the other terms are due to retarded reflections.

For the very first rise, a_h has to be taken into account:

$$a = a_h = \sqrt{\bar{\beta}^2 s^2 + \bar{a} s^{3/2}} = \bar{\beta} s + \frac{1}{2}\frac{\bar{a}}{\bar{\beta}} s^{1/2} - \frac{1}{8}\frac{\bar{a}^2}{\bar{\beta}^3} + \frac{1}{16}\frac{\bar{a}^3}{\bar{\beta}^5} s^{-1/2} - \cdots \quad .$$

Together with the current of range I we obtain, for the current rise on the counter end,

$$i_1 = S \exp(-\bar{\beta} l_1 s) \exp\left(\frac{\bar{a}^2}{8\bar{\beta}^3} l_1\right) \left[s^{-2} \exp\left(-\frac{\bar{a}}{2\bar{\beta}} l_1 s^{1/2} - \frac{\bar{a}^3}{16\bar{\beta}^5} s^{-1/2} + \cdots \right) \right]$$

The first term represents a delay $t_r = \bar{\beta} l_1 = l_1/c$. In the third term only the first component of the exponent is of importance. The inverse transform of this term can be found from the tables of Erdelyi[1)], and we finally obtain

$$I_1 = 0 \qquad \text{for } 0 \leq t \leq l_1/c$$

$$= S \exp\left(\frac{1}{8}\frac{\bar{a}^2}{\bar{\beta}^3} l_2\right) \left\{ \left[\Delta t + \frac{1}{8}\left(\frac{\bar{a}}{\bar{\beta}} l_1\right)^2 \right] \operatorname{Erfc}\left(\frac{1}{4}\frac{\bar{a}}{\bar{\beta}} l_1 \Delta t^{-1/2}\right) \right.$$

$$\left. - \frac{\pi^{-1/2}}{2} \frac{\bar{a}}{\bar{\beta}} l_1 \Delta t^{1/2} \exp\left[-\frac{1}{16}\left(\frac{\bar{a}}{\bar{\beta}}\right)^2 l_1^2 \Delta t^{-1} \right] \right\} \qquad \text{for } \Delta t \lesssim \mu_1 \sigma r_1^2$$

where $\Delta t = t - l_1/c$.

Considering larger t's, the above result has to be complemented by taking into account a_l. A rigorous solution of this case can be found from the literature[2], so that only the result will be represented:

$$I_1 = 0 \qquad \text{for } 0 \leq t \leq \beta l_1$$

$$= S\exp\left(-\frac{a}{2\beta}l_1\right)\left[\Delta t + \frac{a}{2\beta}l_1\int_0^{\Delta t}(\Delta t - \tau)\exp\left(-\frac{a}{2\beta^2}\tau\right)\frac{I_1\left(a/2\beta^2\sqrt{\tau^2 + 2\beta l_1\tau}\right)}{\sqrt{\tau^2 + 2\beta l_1\tau}}\,d\tau\right] \qquad \text{for } \Delta t \gtrsim \mu_1 \sigma r_1^2$$

where $\Delta t = t - \beta l_1$ (I_1 is the modified Bessel function).

Figure 8 shows several examples which have been calculated from approximative expressions of the last two formulae (see the Appendix).

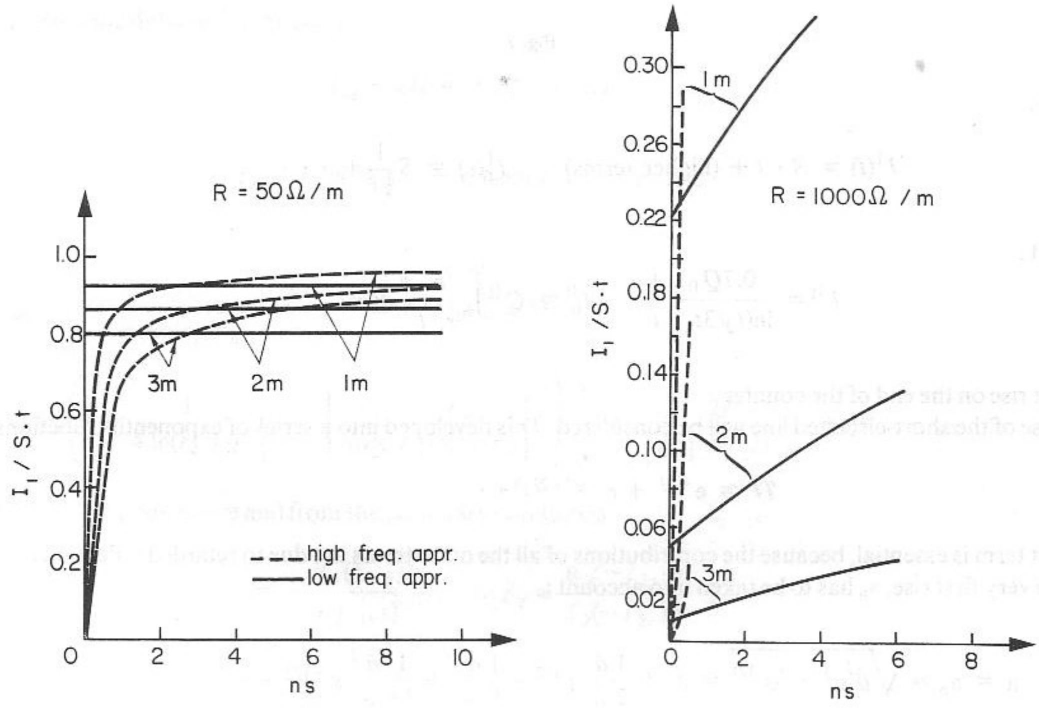


Fig. 8

2.3 Collected charge on the end of the counter

Interesting results are obtained without restrictions on Tr for $t \to \infty$.

The Laplace transform of the charge is given by:

$$q_1 = \frac{Tr_1 i_0}{s} ,$$

from which it follows that

$$\underset{t \to \infty}{Q_1(t)} = \underset{s \to 0}{s q_1} = Tr_1 i_0 \ .$$

Furthermore:

$$\underset{s \to 0}{i_0(s)} = \underset{t \to \infty}{Q_0(t)} \ .$$

Therefore

$$\underset{t \to \infty}{Q_1(t)} = \underset{s \to 0}{Tr(s) Q_0} \quad .$$

Essentially three types of terminations might occur asymptotically :

1) $$\underset{s \to 0}{z_\nu = R_\nu} \quad \text{(asymptotically ohmic)}$$

$$Q_1 = \frac{l_2 + (R_2/R)}{l_1 + l_2 + (R_1 + R_2)/R} Q_0$$

especially for $R_1 = R_2 = 0$

$$Q_1 = \frac{l_2}{l_1 + l_2} Q_0 \quad .$$

2) $$z_\nu = sL_\nu \quad \text{(asympotically inductive)}$$

$$Q_1 = \frac{l_2}{l_1 + l_2} Q_0 \quad .$$

3) $$z_\nu = 1/(sC_\nu) \quad \text{(asymptotically capacitive)}$$

$$Q_1 = \frac{C_1}{C_1 + C_2 + C_0(l_1 + l_2)} \quad .$$

2.4 Asymptotic development for large t's

An analytic treatment is only possible with a limited effort for the case $z_\nu = 0$.
The function

$$Tr_1 = \frac{\sinh\left(\sqrt{as + \beta^2 s^2}\, l_2\right)}{\sinh\left(\sqrt{as + \beta^2 s^2}\, l\right)}$$

has no branch points and only poles of the first order at

$$s_1 \equiv \begin{bmatrix} x_1 = -\dfrac{a}{2\beta^2}\left(1 \pm \sqrt{1 - \dfrac{4\beta^2}{a^2}\left(\dfrac{\pi n}{l}\right)^2}\right) \\ y_1 = 0 \end{bmatrix} \qquad \text{if } 1 \geq \frac{4\beta^2}{a^2}\left(\frac{\pi n}{l}\right)^2$$

and

$$s_2 \equiv \begin{bmatrix} x_2 = -\dfrac{a}{2\beta^2} \\ y_2 = \pm\dfrac{a}{2\beta^2}\sqrt{\dfrac{4\beta^2}{a^2}\left(\dfrac{\pi n}{l}\right)^2 - 1} \end{bmatrix} \qquad \text{if } 1 \leq \frac{4\beta^2}{a^2}\left(\frac{\pi n}{l}\right)^2 \quad ,$$

($n \equiv$ natural number).

The pole (or poles) for which x is largest determines the asymptotic behaviour.
We then have to solve

$$q_1 = \frac{\sinh\left(\sqrt{\beta^2 s^2 + as}\, l_2\right)}{\sinh\left(\sqrt{\beta^2 s^2 + as}\, l\right)} \frac{1}{s} \times i_0(s) = f \times d \quad .$$

The inversion will be executed for the two terms separately, and the final result will be obtained by "Faltung":

$$L^{-1}(f) = \frac{l_2}{l} + \sum_{1}^{\infty} (-1)^n \frac{\sin \pi n(l_2/l)}{\pi n + (al^2/2\pi n)s_i}\, e^{s_i t}$$

where s_i is a pole.
The inversion of the second term simply yields:

$$L^{-1}(d) = I_0(t) .$$

2.4.1 The high-ohmic case

For this case, s_1 should be represented by at least three values, i.e.

$$\left(\frac{a}{2\beta}\frac{l}{\pi}\right)^2 \geq 10 \quad .$$

In the sum of f it suffices to take into account the first term only, and we obtain:

$$L^{-1}(f) = \frac{l_2}{l} - \frac{2}{\pi} \sin \pi \frac{l_2}{l}\, e^{-\delta t} \quad ,$$

with

$$\delta = \frac{1}{a}\left(\frac{\pi}{l}\right)^2 .$$

The "Faltungs" integral for range I (see Fig. 7) can be written as

$$\begin{aligned} Q_1^{\mathrm{I}} &= \frac{l_2}{l}\int_0^{3t_e} I^{\mathrm{I}}(\tau)\mathrm{d}\tau - \frac{2}{\pi}\sin \pi \frac{l_2}{l}\, e^{-\delta t}\int_{3t_e}^{t} e^{\delta\tau} I^{\mathrm{I}}(\tau)\,\mathrm{d}\tau \\ &= \frac{l_2}{l} Q^{\mathrm{I}} - \varepsilon_1^{\mathrm{I}} \end{aligned}$$

$$\varepsilon_1^{\mathrm{I}} < \frac{2}{\pi} \sin \pi \frac{l_2}{l}\, e^{-\delta t}\, 3t_e = \frac{2.4}{\pi} Q_0 \sin \pi \frac{l_2}{l}\, e^{-\delta t} \quad .$$

and for range II,

$$\begin{aligned} Q_1^{\mathrm{II}} &= \frac{l_2}{l} Q^{\mathrm{II}} \int_{3t_e}^{t} \frac{\mathrm{d}\tau}{\tau} - Q^{\mathrm{II}} \frac{2}{\pi} \sin \pi \frac{l_2}{l}\, e^{-\delta t} \int_{3t_e}^{t} \frac{e^{\delta\tau}}{\tau}\,\mathrm{d}\tau \\ &= Q^{\mathrm{II}} \frac{l_2}{l} \ln \frac{t}{3t_e} - \varepsilon_1^{\mathrm{II}} \end{aligned}$$

$$\int_{3t_e}^{t} \frac{e^{\delta\tau}}{\tau}\,\mathrm{d}\tau = \mathrm{Ei}(\delta t) - \mathrm{Ei}(3\delta t_e) \quad .$$

If $t \gg 3t_e$ and $\delta t > 10$, we can use the approximation

$$\mathrm{Ei}(3\delta t_e) \ll \mathrm{Ei}(\delta t) \approx \frac{e^{\delta t}}{\delta t} \quad .$$

Thus

$$\varepsilon_1^{II} = Q^{II}\frac{2}{\pi}\sin\left(\pi\frac{l_2}{l}\right)\frac{1}{\delta t} = \frac{2}{\pi}\sin\left(\pi\frac{l_2}{l}\right)\frac{al^2}{\pi^2 t}Q^{II} \quad ,$$

and it should be noted that $\varepsilon_1^{I} \ll \varepsilon_1^{II}$.

Referring to Fig. 7, we can put $Q^{I} = 0.3\,Q_0$ and $Q^{II} = 0.7\,Q_0$.
Then the collected charge can be written as

$$Q_1 = \left\{\left[0.3 + 0.7\frac{\ln(t/3t_e)}{\ln(t_2/3t_e)}\right]\frac{l_2}{l} - \frac{0.7}{\pi^3\ln(t_2/3t_e)}\frac{al^2}{t}\sin\pi\frac{l_2}{l}\right\}Q_0$$

$$= \left[(1+K_1)\frac{l_2}{l} - K_2\frac{\sin\pi(l_2/l)}{t}\right]0.3Q_0$$

where

$$K_1 = \frac{7\ln(t/3t_e)}{3\ln(t_2/3t_e)} \quad ; \quad K_2 = \frac{7RC_0 l^2}{3\pi^3\ln(t_2/3t_e)}$$

(with a from page 6 and Fig. 5).
The fraction of charge becomes

$$\frac{Q_1}{Q_1 + Q_2} = \frac{\dfrac{l_2}{l} - \dfrac{K_2}{(1+K_1)t}\sin\pi\dfrac{l_2}{l}}{1 - \dfrac{2K_2}{(1+K_1)t}\sin\pi\dfrac{l_2}{l}}$$

Example:

$R = 10^3\,\Omega/\mathrm{m}$; $C_0 = 10^{-11}\,\mathrm{F/m}$; $l = 6\,\mathrm{m}$;
$t_e = 0.2\,\mu s$; $t_2 = 1\,\mathrm{ms}$

$K_2 = 3.6 \times 10^{-9}$ (s)
$K_1 = 0.16$ for $t = 1\,\mu s$
$= 0.66$ for $t = 5\,\mu s$

The evaluation of the charge-distance relation for this example is represented in Fig. 9.

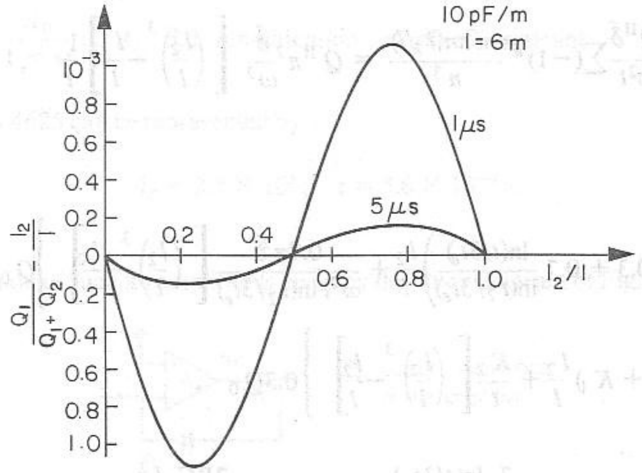


Fig. 9

2.4.2 *The low-ohmic case*

Here it is assumed that

$$\left(\frac{2\pi\beta}{la}\right)^2 \geq 10 \quad .$$

Thus

$$L^{-1}(f) = \frac{l_2}{l} + 2\sum_1^{\infty}(-1)^n \frac{\sin \pi n(l_2/l)}{\pi n} \mathrm{e}^{-\bar{\delta} t}\cos n\omega t$$

where $\bar{\delta} = a/2\beta^2$ and $\omega = \pi/l\beta$.

For range I we obtain

$$Q_1^{\mathrm{I}} = \frac{l_2}{l}\int_0^{3t_e} I^{\mathrm{I}}(\tau)\mathrm{d}\tau + 2\,\mathrm{e}^{-\bar{\delta} t}\sum_1^{\infty}(-1)^n \frac{\sin \pi n(l_2/l)}{\pi n}\int_0^{3t_e} \mathrm{e}^{\bar{\delta}\tau} I^{\mathrm{I}}(\tau)\cos n\omega\tau\,\mathrm{d}\tau$$

$$= \frac{l_2}{l} Q^{\mathrm{I}} + \varepsilon_1^{\mathrm{I}}$$

$$\varepsilon_1^{\mathrm{I}} \ll 2.4 Q_0 \mathrm{e}^{-\bar{\delta} t}\sum_1^{\infty}\frac{\sin \pi n(l_2/l)}{\pi n} \leq 1.2 Q_0 \mathrm{e}^{-\bar{\delta} t} \quad ,$$

and for range II

$$Q_1^{\mathrm{II}} = \frac{l_2}{l} Q^{\mathrm{II}}\int_{3t_e}^{t}\frac{\mathrm{d}\tau}{\tau} + 2Q^{\mathrm{II}}\sum_1^{\infty}(-1)^n\frac{\sin \pi n(l_2/l)}{\pi n}\underbrace{\int_0^{t-3t_e}\frac{\mathrm{e}^{-\bar{\delta}\tau}\cos n\omega\tau}{t-\tau}\mathrm{d}\tau}_{\mathrm{I}}$$

$$= \frac{l_2}{l} Q^{\mathrm{II}} \ln\frac{t}{3t_e} + \varepsilon_1^{\mathrm{II}} \quad .$$

Because $(t-\tau)^{-1}$ is a slowly varying function compared with the trigonometric one, the integral may be approximated by

$$\mathrm{I} = \frac{\bar{\delta}^2}{n^2\omega^2}\int_0^t \frac{\mathrm{e}^{-\bar{\delta}\tau}}{t-\tau} = \frac{\bar{\delta}}{n^2\omega^2 t}$$

for $\bar{\delta} \ll \omega$ and $\bar{\delta}t \geq 10$, so that

$$\varepsilon_1^{\mathrm{II}} = \frac{2Q^{\mathrm{II}}\bar{\delta}}{\pi\omega^2 t}\sum(-1)^n\frac{\sin \pi n(l_2/l)}{n^3} = Q^{\mathrm{II}}\pi^2\frac{\bar{\delta}}{\omega^2}\left[\left(\frac{l_2}{l}\right)^3 - \frac{l_2}{l}\right]\frac{1}{t} \quad ,$$

and finally

$$Q_1 = \left\{\left(0.3 + 0.7\frac{\ln(t/3t_e)}{\ln(t_2/3t_e)}\right)\frac{l_2}{l} + \frac{0.7\pi^2\bar{\delta}}{\omega^2 t\ln(t_2/3t_e)}\left[\left(\frac{l_2}{l}\right)^3 - \frac{l_2}{l}\right]\right\}Q_0$$

$$= \left\{(1+K_1)\frac{l_2}{l} + \frac{K_2}{t}\left[\left(\frac{l_2}{l}\right)^3 - \frac{l_2}{l}\right]\right\}0.3Q_0 \quad ,$$

$$\text{where : } K_1 = \frac{7}{3}\frac{\ln(t/3t_e)}{\ln(t_2/3t_e)} \quad ; \quad K_2 = \frac{7RC_0 l^2}{6\ln(t_2/3t_e)} \quad .$$

The fraction of charge becomes

$$\frac{Q_1}{Q_1 + Q_2} = \frac{\frac{l_2}{l} - \frac{K_2}{(1+K_1)t}\sin\pi\frac{l_2}{l}}{1 - \frac{2K_2}{(1+K_1)t}\sin\pi\frac{l_2}{l}}$$

Example:

$R = 50\,\Omega/\mathrm{m}$; $C_0 = 10^{-11}\,\mathrm{F/m}$; $l = 6\,\mathrm{m}$;
$t_e = 0.2\,\mu\mathrm{s}$; $t_2 = 1\,\mathrm{ms}$

$K_2 = 2.8 \times 10^{-9}\,(\mathrm{s})$
$K_1 = 0.16$ for $t = 1\,\mu\mathrm{s}$
$= 0.66$ for $t = 5\,\mu\mathrm{s}$

The evaluation of this charge-distance relation is represented in Fig. 10.

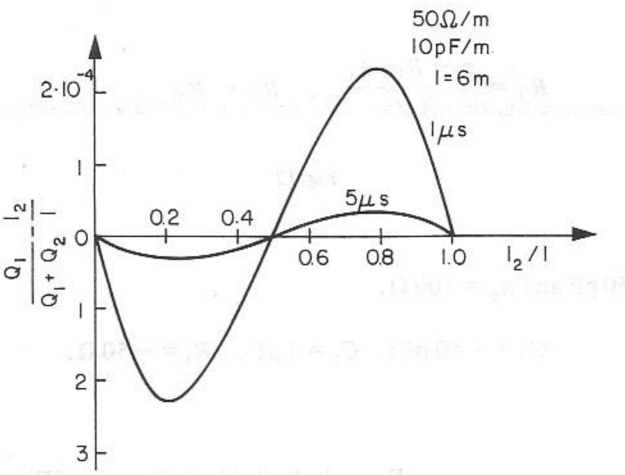


Fig. 10

3. INPUT IMPEDANCE OF CHARGE AND CURRENT AMPLIFIERS

Currents or charges originating from a source of finite internal impedance can be measured with sufficient accuracy only if the input impedance of the measuring device is sufficiently stable or sufficiently small. Operational amplifiers suffer from an inherent frequency-dependent amplification and therefore a constant input impedance cannot be expected. The following approximation, which is rather good for mass-produced operational amplifiers and up to 20 MHz, can be used (in Laplace transform notation):

$$A = \frac{A_0}{1 + s\tau}; \quad A_0: \text{d.c. amplification}, \ \tau: \text{roll-off constant}.$$

The operational amplifier HA 4625 can be represented by

$$A_0 = 2.5 \times 10^5; \quad \tau = 5.6 \times 10^{-4}\,\mathrm{s}.$$

3.1 Charge amplifier

The most simplified version of such an amplifier is given by the block diagram of Fig. 11.

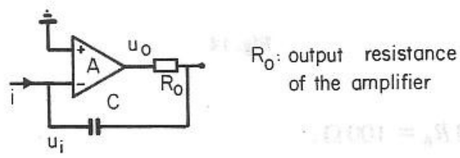


Fig. 11

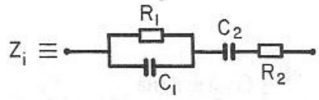
The evaluation of the appropriate relations is so simple that only the results are given (in Laplace transform notation):

$$u_0 = \frac{R_0 A_0}{\tau} \frac{s + (1/R_0 C)}{s[s + (A_0/\tau)]} i \quad ; \quad u_i = -R_0 \frac{[s + (1/R_0 C)][s + (1/\tau)]}{s[s + (A_0/\tau)]} i \quad ;$$

and the input impedance,

$$z_i = -\frac{u_i}{i} = R_0 \frac{[s + (1/R_0 C)][s + (1/\tau)]}{s[s + (A_0/\tau)]} \quad .$$

Its equivalent circuit can be represented by Fig. 12.



$$C_1 = \frac{C\tau}{\tau - CR_0 A_0} \quad ; \quad C_2 = A_0 C$$

$$R_1 = \frac{\tau - R_0 C A_0}{A_0 C} \quad ; \quad R_2 = R_0$$

Fig. 12

For HA 4625, with $C = 50$ pF and $R_0 = 100\,\Omega$,

$$C_1 = -50\,\text{pF}; \quad C_2 = 1\,\mu\text{F}; \quad R_1 = -50\,\Omega.$$

3.2 Current amplifier

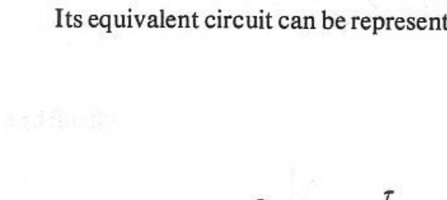


Fig. 13

From the basic block diagram of Fig. 13, it follows simply that

$$u_0 = \frac{RA_0}{\tau} \frac{1}{s + (A_0/\tau)} i \quad , \quad u_i = -R \frac{s + (1/\tau)}{s + (A_0/\tau)} i \quad ;$$

where $R = R_f + R_0$. The input impedance becomes:

$$z_i = R \frac{s + (1/\tau)}{s + (A_0/\tau)}$$

Its equivalent circuit can be represented by Fig. 14.

$$C = -\frac{\tau}{(A_0 - 1)R} \quad ; \quad R_1 = -R\left(1 - \frac{1}{A_0}\right) \quad ; \quad R_2 = R$$

Fig. 14

For HA 4625 with $R_f = 10\,\text{k}\Omega$ and $R_0 = 100\,\Omega$,

$$C \approx -0.2\,\text{pF}; \quad R_1 \approx -10^4\,\Omega.$$

From Fig. 12 and the subsequent example it can be seen that a charge amplifier has a rather complex input impedance, where the values of its components depend on the parameters of the amplifier. The input impedance becomes asymptotically capacitive, so that a precise charge division is not possible.

However in the case of the current amplifier, according to Fig. 14, the input impedance is essentially ohmic, namely $Z_i = R_f/A_0$ (in our example $\simeq 4 \times 10^{-2}\ \Omega$). A line of 100 Ω or more is practically short-circuited, and the precision of charge division is not affected by the input impedance of the amplifier.

4. CONCLUSION

Owing to the dynamics of propagation, the precision of charge division is roughly proportional to the square of the wire length divided by the sampling delay (see for instance p. 11). Over a wide range of resistivity of the wire, this error can be kept small compared with other limiting factors (e.g. uniformity of the wire resistance).

Preference should be given to a low-ohmic wire if simultaneously a drift-time measurement is required (see Fig. 8). However, decreasing resistance provokes an increase in noise so that for each case a compromise has to be found.

The charge measurement is seriously limited by the input impedance (and its fluctuations) of the amplifiers. The input stage of the amplifier should be a current amplifier, and the sampling delay should amount to several microseconds.

With these precautions, the relative errors, resulting from the propagation of the current pulse and from the imperfections of the amplifiers, can be kept much smaller than 10^{-3}.

Acknowledgement

The author would like to express his gratitude to Dr. A. Wetherell for his critical reading of the paper.

APPENDIX

1. *Current rise (high-frequency approximation)*

The Erfc of page 7 is substituted by a polynomial approximation[3]:

$$\mathrm{Erf}\,\theta = 1 - (a_1\Theta + a_2\Theta^2 + a_3\Theta^3)\cdot \exp(-\theta^2).$$

Herewith one obtains

$$I_1 = S\Delta t\left\{(1-2\theta^2)g(\theta) - 2\pi^{-1/2}\theta\right\}\exp\left(-\theta^2 + \frac{\bar{a}^2}{8\bar{\beta}^3}l_1\right) \quad ,$$

where

$$\theta = \frac{\bar{a}}{4\bar{\beta}}\,l_t t^{-1/2}$$

$$g(\theta) = a_1\Theta + a_2\Theta^2 + a_3\Theta^3$$

$$\Theta = (1 + 0.470470\theta)^{-1}$$

$$a_1 = 0.34802\,; \quad a_2 = -0.09587\,; \quad a_3 = 0.74785$$

2. *Current rise (low-frequency approximation)*

The modified Bessel function of page 8 is replaced by a polynomial approximation[4]:

$$I_1(x) = x(0.5 + 6.2499\times 10^{-2}x^2 + 2.6042\times 10^{-3}x^4 + 5.4244\times 10^{-5}x^6)$$
$$x \leq 3.5$$

Using these formulae, the integral may be evaluated even with a programmable pocket calculator.

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1) A. Erdelyi (ed.), Tables of integral transforms (McGraw-Hill, New York, 1954), Vol. I, p. 245.
2) G. Doetsch, Anleitungen zum praktischen Gebrauch der Laplace-Transformation (Oldenburg, Munich, 1956), p. 182.
3) M. Abramowitz and I.A. Stegun (eds.), Handbook of Mathematical Functions (Dover, New York, 1965), p. 299.
4) Ibid., p. 378.

We would like to refer the reader to: V. Radeka, Charge dividing mechanism on resistive electrode in position-sensitive detectors, BNL 25070 (1978).

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THE DYNAMICAL PROPERTIES OF CHARGE-DIVIDING PROPORTIONAL COUNTERS

p. 7 The last formula on this page should be replaced by:

$$
\begin{aligned}
I_1 &= 0 \qquad\qquad \text{for } 0 \leq t \leq l_1/c \\
&= S \exp\left(\frac{1}{8}\frac{\bar{a}^2}{\bar{\beta}^3} l_1\right) \left\{ \left[\Delta t + \frac{1}{8}\left(\frac{\bar{a}}{\bar{\beta}}\, l_1\right)^2\right] \mathrm{Erfc}\left(\frac{1}{4}\frac{\bar{a}}{\bar{\beta}} l_1 \Delta t^{-1/2}\right) \right. \\
&\quad \left. - \frac{\pi^{-1/2}}{2}\frac{\bar{a}}{\bar{\beta}}\, l_1 \Delta t^{1/2} \exp\left[-\frac{1}{16}\left(\frac{\bar{a}}{\bar{\beta}}\right)^2 l_1^2 \Delta t^{-1}\right]\right\} \qquad \text{for } \Delta t \lesssim \mu_1 \sigma r_1^2
\end{aligned}
$$

p. 8 The first formula on this page should be replaced by:

$$
\begin{aligned}
I_1 &= 0 \qquad\qquad \text{for } 0 \leq t \leq \beta l_1 \\
&= S \exp\left(-\frac{a}{2\beta} l_1\right)\left[\Delta t + \frac{a}{2\beta} l_1 \int_0^{\Delta t} (\Delta t - \tau) \exp\left(-\frac{a}{2\beta^2}\tau\right) \frac{\mathrm{I}_1\left(a/2\beta^2\sqrt{\tau^2 + 2\beta l_1 \tau}\right)}{\sqrt{\tau^2 + 2\beta l_1 \tau}}\, d\tau\right] \qquad \text{for } \Delta t \gtrsim \mu_1 \sigma r_1^2
\end{aligned}
$$

p. 13 The formula at the top of this page should be replaced by:

The fraction of charge becomes

$$
\frac{Q_1}{Q_1 + Q_2} = \frac{\dfrac{l_2}{l} + \dfrac{K_2}{(1 + K_1)t}\left[\left(\dfrac{l_2}{l}\right)^3 - \dfrac{l_2}{l}\right]}{1 + \dfrac{K_2}{(1 + K_1)t}\left[\left(\dfrac{l_2}{l}\right)^3 + \left(\dfrac{l - l_2}{l}\right)^3 - 1\right]}
$$

p. 16 The second formula should be replaced by:

$$
I_1 = S\Delta t \left|(1 + 2\theta^2) g(\theta) - 2\pi^{-1/2}\theta\right| \exp\left(-\theta^2 + \frac{\bar{a}^2}{8\bar{\beta}^3} l_1\right) \quad ,
$$

where

$$
\theta = \frac{\bar{a}}{4\bar{\beta}}\, l_1 \Delta t^{-1/2}
$$

July 1982

Dr. Fritz Schneider (CERN) 1983, genannt "Emmesse Fritz"
*1928 Rodenbach/KL, Auf den Felsen
+1989 Stahlberg/Rockenhausen
Er wurde 61 Jahre alt.

Er misst sich grad geännert hann!

Heit Meddag trefft mich faschd de Blitz:
"E Verlobungskaard vum Schneider Fritz",
kreischd mer mei Fraa ins rechde Ohr.
Ich denk zuerschd, es is net wohr
doch wie ich a mei Au uffreiß,
do steht es vor mer. Schwarz uff weiß
so deidlich, daß mers glaawe kann -
Er misst sich grad geännert hann!

Wie konnt dann sowas nor bassiere?
Viel leichter kammer Flöh´ dressiere,
wie so e Mann an sich zu binne.
Jetzt san mer mol, wie konnts gelinge?
Das hann ich werklich net verstann -
Er misst sich grad geännert hann!

Des Mädche deht mich intressiere
wo so e Kunschdstück konnt vollfiehre.
Ei, demm sei Blut muß häämer sin
wie Lava innem Ätna drin,
schunn wär der Eisberg net geschmolz!
(Dodruff is "Sie" bestimmt a stolz!)
No so em Junggesellelewe
muß der im siebte Himmel schwewe.
Dem werd die Zeit bestimmt zu lang -
Er misst sich grad geännert hann!

Wie ich en im Gedächtnis hann
war das e ruhiger, stiller Mann.
Der wollt doch vunn de Määd nix wisse.
(Ich glaab, noch netemol vumm Küsse)
Is niemols in e Wertschaft gang,
trinkt Wasser schunn sei Lewe lang.
Hat Wert geleet uff sei Frisur
unn uff sei Adonis-Puschdur,
hot immer Handschu an de Henn
grad wie e richt´ger Tschentlemenn.
Kurzum, er war e feiner Mann -
   Er misst sich grad geännert hann!

Unn was des for e Baschdler war?
Vunn dem, was annere fortgefahr,
aus roschdige Schrauwe, Kabelende
baut der die dollschde Instrumente.
Der baut aus Mickedroht unn Zinn
e Fernsehtruh´mit Hausbar drinn.
In Genf, hann neilich ich geheert
hat er e Maschinsche vorgefehrt,
des macht unner Annerem newebei
de Schweizer Kees trichinefrei!
Unn was der sunschd noch alles kann -
   Er misst sich grad geännert hann!

Unn singe konnt er, wunnerschä,
zwä- dreistimmisch als ganz allä
veel besser als der Elvis Preslich,
dem sei Gesang der is jo gräßlich.
Duht seller Mensch es Maul uffreiße
bringt der e Schnellzug zum entgleise.
Dem sei Gesang is doch e Graus;
der zieht em jo die Socke aus.
Ganz annerschd war dem Fritz sei G´sang -
Er misst sich grad geännert hann!

Faß´ ich´s zusamme jetzt am End:
Er is e vielseitig Talent!
Dann was er duht unn was er schafft
is werklich alles "sagenhaft".
Des gebt e gurer Ehemann
do hann ich garkää Zweifel dran.
Als Freind muß ich en jo gut kenne!
(Ich darf mich hoffendlich so nenne?)
Unn wann Ehr mol verheirat´ sinn
unn kriejen Eier erschdes Kind
mell ich mich schunn als Pate jetzt,
unn hoff, die Stell is net schunn b´setzt.
Met Kinner war er immer schunn ganz närrisch
e Sticker viere hat er sich gewinscht - orre err ich?
Ich glaab, ich hatt´ en richtig doch verstann -
Er misst sich grad geännert hann!

Edgar Scheuermann

Weitere Arbeiten (u.a.):

32263 (CERN-PS/FS-3) PULSED PROTON SOURCE FOR HIGH CURRENT. [Fritz Schneider] (European Organization for Nuclear Research, Geneva). Jan. 1957. 6p.

The parameters involved in the design of a pulsed proton source for high currents are considered. A proton source is described with the following characteristics: r-f pulse time, 250 μsec; extraction time, 10 μsec; and cup current at 3 cm behind canal, 150 ma. (D.L.C.)