Energy-Range Formula for Electro Flight

Kapt. Wolf Scheuermann

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1 Introduction

The task of this document is to find an analogon to the Breguet-formula [1] for electric powered flight. While for the conventional powered aircraft mass effects due to fuel burn play a distinctive role for range, this is not the case in electric flight. Here, only the amount of energy that can be stored and used, the aircraft mass, and the aerodynamic properties of the aircraft come into calculation.

While gliding an aircraft transforms its potential energy of its initial altitude into velocity, range, and loss of height. To keep the altitude in level flight the equivalent of stored energy to the potential energy has to be used to get the range.

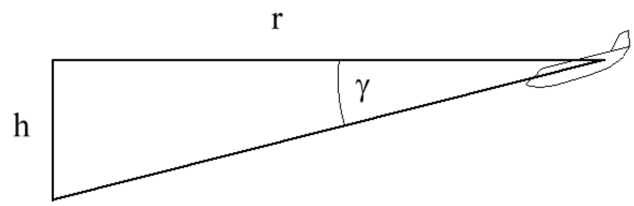

Figure 1: Gliding vs Level Flight

2 Range as Function of Energy

During glide flight the potential energy is $E_{pot} = m \cdot g \cdot h$ due to loss of height h. Here, m is the mass of the aircraft and $g = 9.81\frac{m}{s^2}$ the gravitational acceleration of the earth, simplifying assumed to be constant.

An electric powered aircraft has no mass loss due to fuel burn contrary to conventional motorized flight.

The aerodynamic properties of the aircraft are expressed by the Glide Number $G = \frac{c_L}{c_D}$, where c_L is the lift coefficient and c_D is the drag coefficient.

The kinetic energy $E_{kin} = \frac{1}{2}mv^2$ must be equal to E_{pot} or the carried energy E.

The glide slope γ is given by $tan(\gamma) = \frac{1}{G} = \frac{c_D}{c_L} = \frac{h}{r}$.This allows to solve for the range r.

$r = h \cdot \frac{c_L}{c_D} = h \cdot G$

From $E_{pot} = E = m \cdot g \cdot h$ follows $h = \frac{E}{mg} = \frac{E_{kin}}{mg} = \frac{1}{2} \cdot \frac{mv^2}{mg} = \frac{1}{2} \cdot \frac{v^2}{g}$.

From $E_{kin} = E = \frac{1}{2}mv^2$ follows $v^2 = \frac{2E}{m}$

Therefore $h = \frac{1}{2}\frac{v^2}{g} = \frac{1}{2}\frac{2E}{mg} = \frac{E}{mg}$

So, the range as a function of energy, mass, and glide number is given by

$$r = \frac{E \cdot G}{m \cdot g}$$

where the units are: $[\frac{kg \cdot \frac{m}{s^2} \cdot m \cdot 1}{kg \cdot \frac{m}{s^2}}] = [m]$

3 Example

The maximum range of the E-Motor powered Flying Wing ScV13e is calculated as an example.
The following are the data of this aircraft design:

G = 10 conservative guess for best glide (max range) or min power required (max endurance)
m = 300 kg
g = 9.81 $\frac{m}{s^2}$
$E_{Akku} = 400V \cdot 100Ah \cdot 3600\frac{s}{h} = 144MJ$ (such batteries are not yet existent!)
$\eta_{EMotor} = 0.8$
$\eta_{Impeller} = 0.8$
$\eta = \eta_{EMotor} \cdot \eta_{Impeller} = 0.6$
$E = E_{Akku} \cdot \eta = 86MJ$

With these data the maximum range is given by

$r = \frac{E \cdot G}{m \cdot g} = \frac{86 \cdot 10^6 \cdot 10}{300 \cdot 9.81} [m] = 292 km = 158 NM$

This range corresponds quite well to the conservative guesses of the Aircraft Operating Manual of this aircraft [2].

4 Acknowledgement

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5 Literature

References

[1] G. Brüning, X. Hafer, G. Sachs, Flugleistungen Springer Verlag, 3.Auflg., Berlin/Heidelberg 2006

[2] Wolf Scheuermann, Aircraft Operating and Flight Manual v 1.0, Experimental Flying Wing ScV13e www.forschungskontor.de, Hamburg 2012